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AN ALTERNATIVE APPROACH TO THE VARIABLE HOUSING ALLOWANCE
PROGRAM

(94 pages-including title page and table of contents)

Gregory Ronald Gerth
Captain, U.S. Air Force

Master of Arts

Economics

1987

University of Virginia

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ABSTRACT

↖ This thesis discusses the essential features of an alternative to the current variable housing allowance (VHA) program, which is called the variable compensation program. The outline of the variable compensation program begins by assuming a typical servicemember has a Stone-Geary utility function. Next, it derives this typical servicemember's implied expenditure function. This expenditure function is a function of the prices of two composite commodities, housing and nonhousing goods and services, and the level of utility provided by the member's regular military compensation. The parameters of this function are the respective subsistence quantities and marginal budget shares of the two composite goods. The subsequent sections of the variable compensation program outline discuss: (1) techniques to estimate these parameters, (2) the methodology of determining the appropriate levels of utility for the various military paygrades, (3) techniques to estimate the geographic price levels of housing and nonhousing goods and services, and (4) potential sources of data required by this program. This thesis concludes with a discussion of the advantages of the variable compensation program over the current VHA program as well as possible areas of expansion to make this program a more sophisticated analysis of military compensation. (Thesis) ↗

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PROGRAM

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Master of Arts

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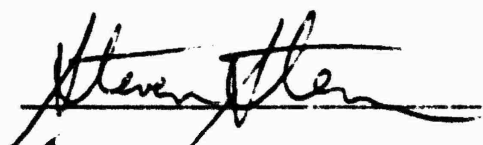
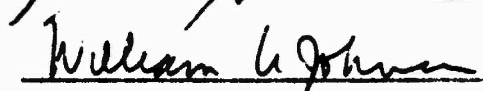



Table of Contents

	Page
1.0 Introduction	1
2.0 Variable Housing Allowance (VHA) Program Background	3
2.1 Footnotes to Section 2	10
3.0 An Alternative Approach to the VHA Program	12
3.1 Selecting the Utility Function	13
3.2 Estimating the Parameters	27
3.3 Determining Utility Levels for the Different Paupgrades	38
3.4 Availability of Data	50
3.5 Footnotes to Section 3	64
4.0 Comparison of the Current VHA Program and the Variable Compensation Program	69
4.1 Footnotes to Section 4	77
5.0 Conclusion	78
6.0 Bibliography	82
Appendix One	88
Appendix Two	90
Appendix Three	92

1.0 INTRODUCTION

The military services of the United States either provide government housing for their members or pay them a basic allowance for quarters (BAQ) to assist them in obtaining private off-base housing. These BAQ payments vary according to the member's rank and whether or not the member has any dependents. Details of BAQ entitlement are defined in Section 403 of Title 37 of the United States Code (37 U.S.C. 403). Section 4(c) of Public Law 96-343-September 8, 1980 amended 37 U.S.C. 403 to provide a supplemental variable housing allowance (VHA) to those servicemembers currently receiving BAQ who reside in a high housing cost area of the United States. Congressional rationale for implementing the VHA program was to try and make military compensation for a paygrade roughly the same regardless of assignment location.

Section two of this study provides an overview of the current structure of the VHA program. Section three introduces an alternative program concept which I call the variable compensation program. The variable compensation program is designed to provide servicemembers of the same paygrade equal compensation regardless of assignment location. This program assumes a Stone-Gear utility function for a typical

servicemember. Then, using the implied expenditure function with different utility levels for different paygrades and geographic price data, this program can determine appropriate total compensation levels for every paygrade at each military location area. These compensation levels will make the utility of servicemembers in each paygrade independent of their assignment locations. Section four concludes with a comparison of the existing VHA program and the proposed variable compensation program and suggests additional topics that might be considered in a more intricate analysis of military compensation.

I realize this paper suggests an alternative to the VHA program that would completely overhaul the traditional structure of military compensation. However, it is my opinion that the continuous debate and fine tuning of the VHA program by Congress and the Department of Defense reflects dissatisfaction with the program at the highest civilian and military levels. This paper does not produce a model of military compensation ready for immediate implementation. Rather, it is my hope that the alternative ideas I suggest about equitable compensation, regardless of assignment location, might at least offer some insight into improving the current VHA program.

2.0 VARIABLE HOUSING ALLOWANCE PROGRAM BACKGROUND

Military compensation includes many different elements such as base pay, basic allowance for subsistence (BAS), basic allowance for quarters (BAQ), variable housing allowance (VHA), special and incentive pay, retirement pay and benefits such as medical care, commissary and exchange privileges, and others. An important measure of active duty military compensation is called regular military compensation (RMC). RMC consists of base pay, BAS, BAQ, VHA (when applicable) and the federal tax advantage since BAS, BAQ, and VHA are nontaxable allowances. RMC is an important measure because Congress uses it as the measure of a military salary in comparing the adequacy of military pay levels with those in the civilian sector.¹

The VHA is a relatively new component of regular military compensation. Section 403 of Title 37 of the United States Code (37 U.S.C. 403) provides BAQ as an entitlement to military personnel who are not provided government quarters. The amount of BAQ paid to a servicemember depends on the member's rank and dependency status. Section 4(c) of Public Law 96-343 amended 37 U.S.C. 403 to provide a supplemental VHA to members living in high cost areas. Eligible members began receiving their VHAs on October 1, 1980 and the

program became a legal entitlement on September 30, 1981.

The need for the VHA program arose from the substantial differences in the prices of goods and services throughout the United States. Prior to the initiation of the VHA program, all elements of regular military compensation for servicemembers of the same paygrade and dependency status were the same regardless of location of assignment in the continental United States. Thus, a servicemember stationed in a higher cost area suffered a lower standard of living than a member of the same paygrade assigned to a lower cost area. The requirement to move frequently and the inability of the centralized assignment process to "ensure that members spend equal time in low-cost and high-cost areas" created a financial burden which was detrimental to retaining and recruiting military personnel.²

In order to improve retention and recruiting, Congress recognized the need to compensate military personnel for the price differentials of goods and services at different duty locations throughout the United States.³ Since housing prices differ more on a geographical basis than the prices of other goods and services, Congress authorized the VHA program to help

offset the financial burden of assignments to high cost areas.

The current VHA program is structured in the following manner. The Department of Defense (DoD) conducts a biennial sample survey of military personnel's housing expenses. The housing expenses reported by the servicemembers include rent or rental equivalence for homeowners, utilities, insurance, and maintenance expenses. DoD then uses regression analysis to determine the local median housing cost for all twenty-three paygrades at each of the over 340 military location areas in the continental United States, Alaska, and Hawaii.⁴ 37 U.S.C. 403a.(c)(1) defines the current method of calculating VHA rates for servicemembers as follows. A VHA is paid to a servicemember, in addition to a BAO, whenever the local median housing cost of servicemembers with the same paygrade and dependency status exceeds 80 percent of the national median housing cost of servicemembers in that same paygrade and dependency status. However, the VHA is not a cash grant. 37 U.S.C. 403a.(c)(6)(A), effective March 1, 1986, requires that if a military servicemember's total housing allowances (BAO plus VHA) exceed the member's reported housing expenses, then the servicemember must pay back to the government fifty percent of the

difference, not to exceed the total amount of the VHA (no servicemember has to pay back any BAQ).

The following example illustrates two hypothetical cases of VHA calculations. This example is for the officer paygrade O-3 (captain in the Air Force, Army, and Marine Corps - lieutenant in the Navy) and assumes the officer has at least one dependent. LMHC and NMHC stand for local median housing cost and national median housing cost, respectively. All dollar figures except the BAQ (as of January 1, 1986) are hypothetical.

HYPOTHETICAL MONTHLY VHA CALCULATIONS

<u>BAQ</u>	<u>Duty Location</u>	<u>LMHC</u>	<u>80% of NMHC</u>	<u>VHA</u>	<u>Total Allow</u>
\$433.50	Darton, OH	\$550	\$534	\$16	\$449.50
\$433.50	Wash. DC	\$800	\$534	\$266	\$699.50

Thus, this example illustrates the method of calculating the specific VHA (LMHC - .8 NMHC) for the paygrade O-3 (with dependents) in two different duty locations.^a An O-3 stationed in Darton, Ohio will receive the full \$16 of VHA unless the officer's reported housing expenses are less than the total housing allowances of \$449.50. For example, if a particular officer's reported housing expenses were only \$400, this officer would pay back fifty percent of the difference between the total allowances and the reported housing expenses

[.5(\$449.50-\$400)=\$24.75]. not to exceed the VHA amount of \$16. Therefore, in this specific case, the officer would pay back \$16 and receive only the BAQ of \$433.50.

Recent legislation specifically addressed the cost growth and distribution method of the VHA program. 37 U.S.C. 403a.(d)(1) places an upward cap on the total amount of the annual budget for the VHA program. This limit is the ratio of the October military adjusted housing component of the Consumer Price Index (CPI) for the previous fiscal year over the same index for October two fiscal years ago multiplied by the previous fiscal year's VHA budget. For example, the upward cap for the total VHA budget for fiscal year 1987 (October 1, 1986 to September 30, 1987) was the ratio of the military adjusted housing component of the CPI for October 1985 divided by the same index for October 1984 times the fiscal year 1986 VHA budget. Furthermore, effective October 1, 1985, any increase in the VHA budget in an odd-numbered year will be distributed evenly to all paygrades, regardless of assignment location. However, in even-numbered years, any increase in the VHA budget will be distributed in a tailored manner based on the findings of the biennial housing survey.* A higher proportion of the budget increase will go to members stationed in areas of higher cost growth.

With this brief overview of the VHA program, several key points are evident. The current program conducts an expensive sample survey of housing expenses of military personnel biennially. It also requires an even more administratively burdensome annual documentation of housing expenses by the more than 900,000 military members receiving VHA.⁷ Documentation requires a copy of the rental agreement signed by the landlord and the servicemember or a copy of a mortgage payment coupon (DoD estimates utility and maintenance expenses by location). Additionally, the VHA is not a cash grant. Military members must spend their full housing allowances on housing expenses or forfeit fifty percent of the difference up to the total amount of the VHA. Furthermore, the VHA program rates are based on self-reported housing expenditure data, uncontrolled for quality, rather than independent housing price indices. Ironically, implementation of the recent system in which odd-numbered years see VHA budget increases spread uniformly across all paygrades, regardless of assignment location, will hurt those members in the higher cost areas. Their VHA rates will fall further behind their increasing costs of housing than those members stationed in lower cost areas. This problem is precisely what the VHA program was created to prevent. Finally, the VHA program only attempts to compensate military personnel

for price differentials in housing and treats the prices of other goods and services as uniform throughout the United States.

2.1 FOOTNOTES TO SECTION 2

¹ "Pay and Allowances." All Hands, Mar 1986, p. 40: hereafter cited as All Hands.

² A Joint Service Study, Variable Housing Allowance (VHA) Program. Should it be Changed? (Feb. 1984), p. I-2: hereafter cited as VHA Program.

³ Prior to November 8, 1985 all military personnel assigned to Alaska or Hawaii were compensated like all other servicemembers stationed outside the continental United States (CONUS). Since 1946, servicemembers assigned outside the CONUS have received a station housing allowance (authorized in Section 405 of Title 37 U.S.C.) or, as of July 1981, a "rent plus" allowance to compensate them for cost differentials in locations outside the CONUS. However, Public Law 99-145 section 603(b)(2)(A) requires all military personnel reporting for duty in Alaska or Hawaii after November 8, 1985 to be under the VHA program. Servicemembers assigned to Alaska or Hawaii who were receiving "rent plus" allowances prior to November 9, 1985 will continue to be grandfathered under the "rent plus" system until their next permanent change of station or November 7, 1989 (whichever occurs sooner). See VHA Program, p. I-3. Also, see All Hands, p. 44.

⁴ VHA Program, p. II-3.

• Military personnel who are not provided government quarters receive the full BAO even if the local median housing cost is less than 80 percent of the national median housing cost for their paygrade ($VHA = LMHC - .8NMHC$ and $VHA \geq 0$).

• P.J. Budahn. "2.45% Increase in VHA Possible." Air Force Times. 6 Oct. 1986, p. 4.

• P. J. Budahn. "VHA Offset Saves More Than \$69 Million a Year." Air Force Times. 14 Jul. 1986. p.4.

3.0 AN ALTERNATIVE APPROACH TO THE VHA PROGRAM

The current variable housing allowance program attempts to eliminate the differences in standards of housing due to assignment location in the United States. An alternative concept to the variable housing allowance program is a program that recognizes that the prices of goods and services other than housing also vary widely throughout the United States. I will call this alternative concept the variable compensation program.

Designing a variable compensation program that provides equitable compensation to servicemembers of the same paygrade, regardless of assignment location in the United States, requires the following process: Specify a utility function for the typical servicemember. This utility function then implies an expenditure function. Estimate the parameters of this expenditure function. Determine the appropriate utility levels for all paygrades in the military services. Gather available price data on housing and nonhousing goods and services for the more than 340 military localities in the United States. Then, the variable compensation program can use the expenditure function, price data, and appropriate level of utility to calculate the equitable level of paid compensation for a servicemember's paygrade regardless of assignment location in the United States.

3.1 SELECTING THE UTILITY FUNCTION

This section discusses selecting a utility function for a typical servicemember and mathematically deriving his implied expenditure function. It begins by defining the concept of utility and one of its key properties. Given this framework, it then outlines and solves the utility maximization problem facing a typical servicemember by using the Lagrange multiplier method. Applying the concept of duality, it also solves the dual cost minimization problem for a typical servicemember which finally leads to his implied expenditure function.

Servicemembers, like other individuals, consume a variety of goods and services. Consumption of these commodities provides satisfaction, or what economists call utility, to the consumer. Consumption of different bundles of goods and services provides different levels of utility to the consumer. It is also possible that quantities trade-offs within the same bundle of commodities can provide the same level of utility to the consumer. The important point is that a consumer has a preference ranking for the various combinations of the goods and services he consumes, which depends on the levels of utility he derives from the various commodity combinations. This subjective preference ranking of a

consumer is represented by the consumer's utility function. A utility function is expressed as $U(X_1, \dots, X_n)$ where U is utility and X_1, \dots, X_n are the amounts of the n goods and services the individual consumes.

An important property of utility is that it is an ordinal measure. This means simply that a utility function only ranks combinations of consumption goods and services as first, second, third, etc. The utility function does not reveal how much more a consumer prefers one combination to another. Thus, a utility function simply serves as an index of a consumer's preferences. The utility index for a consumer is highest for the most preferred bundle of commodities and decreases down to the least preferred bundle.

Economists assume that a rational consumer will choose the consumption bundle of goods and services that maximizes his utility. However, a consumer is also faced with a limited amount of resources to spend on goods and services known as a budget constraint.

This budget constraint is expressed as $P_1X_1 + \dots + P_nX_n = M$ where P is price, X is the quantity of the commodity consumed for all goods and services $1, \dots, n$ and M is income or total expenditure. Thus, the "classical problem in the theory of the consumer" is stated as:

maximizing $U(X_1, \dots, X_n)$

subject to $P_1X_1 + \dots + P_nX_n = M.$ ²

Therefore, the classical choice problem for a servicemember is like that of any consumer; maximize his utility subject to his budget constraint.

The steps involved in solving the constrained utility maximization problem for a typical servicemember are as follows: (1) Specify assumptions, notation, and formally state the problem. (2) Define the necessary first-order conditions. (3) Define the sufficient second-order conditions. (4) Describe the Lagrange multiplier method and use it to derive the necessary first-order conditions. (5) Solve the necessary first-order equations simultaneously to derive the typical servicemember's demand equations for housing and nonhousing goods and services. (6) Derive the second-order conditions.

The following assumptions are vital to simplifying my analysis. I hope this simplified analysis will still yield some beneficial insight. First, I assume there is a typical servicemember whose preferences are a reasonable representation of the diverse military community. Secondly, I assume that a typical servicemember's utility function is one particular type of the many different types of utility functions that exist in the economics literature. Specifically, I

elect to use a Stone-Geary utility function because it is easy to work with and it maintains its linear estimation properties under aggregation of individual demand equations. Thirdly, I ignore the effects of commissaries, base or post exchanges, military medical benefits, etc. on the servicemember's preferences in the utility function. These effects would greatly complicate the analysis. Finally, I assume that the goods and services military personnel consume fall into one of the following two categories of composite goods: housing or nonhousing goods and services.

Given these assumptions, I apply the following notation. The utility function for a typical servicemember is $U(X_1, X_2) = (X_1 - B_1)^v (X_2 - B_2)^{1-v}$. U is utility. X_1 is the quantity of housing consumed. X_2 is the quantity of nonhousing goods and services consumed. B_1 and B_2 are the subsistence levels of housing and nonhousing goods and services, respectively. Thus, B_1 and B_2 represent the necessary or minimum levels of housing and nonhousing goods and services a servicemember must consume to exist. The terms v and $(1-v)$ are the marginal budget shares of housing and nonhousing goods and services, respectively. Therefore, v represents the marginal propensity of a member to spend additional income on housing. Likewise, $(1-v)$ represents the marginal propensity of a member to spend

additional income on nonhousing goods and services. For example, if $v=.4$ and $1-v=.6$, a member given an additional increase in income of \$100 will spend \$40 more on housing and \$60 more on nonhousing goods and services.

Given this notation, the constrained maximization problem for a typical servicemember is written:

$$\begin{aligned} \text{maximize } U(X_1, X_2) &= (X_1 - B_1)^v (X_2 - B_2)^{1-v} \\ \text{subject to } P_1 X_1 + P_2 X_2 &= M. \end{aligned}$$

The next step in solving this constrained maximization problem is to first define the necessary first-order conditions and sufficient second-order conditions.

The necessary first-order conditions for a constrained maximization problem are defined as those conditions that must hold for a potential maximum value of a function to exist. Even if the necessary first-order conditions are met, the function is only potentially at its maximum value. Additional second-order conditions determine whether a function that meets its necessary first-order conditions is at a maximum, minimum, or an inflection point. Furthermore, since it is a realistic restriction to limit the quantities of X_1 and X_2 to be the set of nonnegative numbers, the terms maximum and minimum in this thesis refer to the relative and not the global extreme points of a function.

Basically, the necessary first-order condition for a function of a single variable to reach its maximum value is that the derivative of the function with respect to its variable must equal zero. Mathematicians define the derivative of a function as the rate of change of a function with respect to a very small change in its variable.² Thus, when the derivative or rate of change of a function equals zero, no small change in its variable can increase the value of the function. Therefore, it is a necessary requirement that the derivative of a function equals zero at the maximum value of the function. If the derivative of the function were not equal to zero, then the function would either be increasing or decreasing. Once again, even if the derivative of the function is zero, the function is still only potentially at its maximum value.

The necessary first-order condition for a function of a single variable only identifies candidates for a maximum value. Because the derivative of the function equals zero, the slope of the function's curve is flat. The next step is to determine whether the function is at the peak of a hill (maximum), the bottom of a valley (minimum), or is at an inflection point. The sufficient second-order condition test utilizes the second derivative of a function. The second derivative of a function is the derivative of the first derivative of

the function. In other words the second derivative is the rate of change of the first derivative. Therefore, if the second derivative of the function is evaluated at a point satisfying the necessary first-order condition and is less than zero; then, the slope of the function's curve decreases with any small movement from that point and the function is at its maximum value. Likewise, if the second derivative is greater than zero, then the slope of the function's curve increases with any small movement from that point and the function is at its minimum value. If the second derivative is equal to zero, then the function may still be at either a maximum, minimum, or an inflection point. An additional test such as the "Nth-derivative test" is required to determine what type of extreme point exists.³

For a function of more than one variable to reach its maximum value, the necessary first-order conditions are that the partial derivatives of the function must all equal zero. The term "partial derivative" is the derivative of a function with respect to one of its variables while all the other variables are held constant. Thus, when all the partial derivatives of a function equal zero, there is no combination of small changes in the variables of the function that will increase its value. However, additional second-order conditions again determine whether the function is at a

maximum, minimum or an inflection point. The appropriate sufficient second-order conditions for a function of more than one variable are more complex and are detailed in a separate appendix for each constrained optimization problem solved in this thesis.

As described above, the necessary first-order conditions to maximize a function simply involve setting the partial derivatives of the function with respect to each of its variables equal to zero. For a function of n variables this results in n equations in n unknowns. The two requirements to solve a system of linear equations are that the number of equations equal the number of unknowns and that the equations are linearly independent. However, in our constrained maximization problem, there is one additional equation (the budget constraint) but no additional variables. In order to add another variable to our problem, we can employ the Lagrange multiplier method. The Lagrangian method involves expressing the utility function, $U(X_1, X_2)$, as a function of an additional independent variable, which is called the Lagrange multiplier, or z in this problem. This is accomplished in the following manner. The utility function is rewritten from

$$U(X_1, X_2) = (X_1 - B_1)^{\alpha}(X_2 - B_2)^{1-\alpha}$$

to

$$L(X_1, X_2, z) = (X_1 - B_1)^{\alpha}(X_2 - B_2)^{1-\alpha} + z(M - P_1X_1 - P_2X_2).$$

This new form of the utility function embodies the budget constraint and is called the Lagrangian function which is represented by the letter L . Taking the derivative of the Lagrangian function L with respect to X_1 , X_2 , and z yields the following necessary first-order conditions (L_1 is the derivative of the function L with respect to X_1 , etc.):

$$L_1 = v(X_1 - B_1)^{v-1}(X_2 - B_2)^{1-v} - zP_1 = 0$$

$$L_2 = (1-v)(X_1 - B_1)^v(X_2 - B_2)^{-v} - zP_2 = 0$$

$$L_z = M - P_1X_1 - P_2X_2 = 0.$$

The condition that $L_z = M - P_1X_1 - P_2X_2 = 0$ guarantees that the servicemember spends his entire income on X_1 and X_2 . Therefore, the term $z(M - P_1X_1 - P_2X_2)$ in the Lagrangian function $L(X_1, X_2, z)$ is zero regardless of the value of z . As a result, the maximum values of the Lagrangian function $L(X_1, X_2, z)$ are the same as those of the utility function $U(X_1, X_2)$ since

$$L(X_1, X_2, z) = U(X_1, X_2) + z(M - P_1X_1 - P_2X_2).$$

Furthermore, the Lagrange multiplier method is more than a useful mathematical device to solve a constrained maximization problem. The Lagrange multiplier z also has an important economic interpretation. Solving the above conditions L_1 and L_2 for z yields:

$$z = [v(X_1 - B_1)^{v-1}(X_2 - B_2)^{1-v}]/P_1,$$

and

$$z = [(1-v)(X_1 - B_1)^v(X_2 - B_2)^{-v}]/P_2.$$

This is the same as $z = U_1/P_1 = U_2/P_2$ (U_1 is the partial derivative of the utility function with respect to X_1 , etc.). Hence, z is the marginal utility of each good divided by its price or the marginal utility of income. Since economists assume a consumer prefers more of any good to less (with all other conditions held equal) the marginal utility of income is positive. Therefore, the Lagrange multiplier in our problem also leads the servicemember to spend his entire income on the only goods available that give him utility.

Given this explanation of the Lagrange multiplier method and the resulting necessary first-order conditions L_1 , L_2 , and L_3 above, the next step is to derive the servicemember's demand equations for housing and nonhousing goods and services. Solving these three necessary first-order conditions ($L_1 = L_2 = L_3 = 0$) simultaneously yields:

$$X_1 = B_1 + v(M - P_1B_1 - P_2B_2)/P_1$$

and

$$X_2 = B_2 + (1 - v)[M - P_1B_1 - P_2B_2]/P_2$$

as the optimal consumption amounts of housing and nonhousing goods and services to maximize utility. The expression for the optimal amount of X_1 states that the servicemember should procure the subsistence amount of housing (B_1) plus the amount of housing that can be bought with the marginal budget share of housing (v)

times the income left after purchasing the subsistence levels of housing and nonhousing goods and services. Likewise, the expression for X_2 states the optimal amount of nonhousing goods and services is the subsistence amount (B_2) plus the amount that can be bought with the marginal budget share of nonhousing goods and services ($1-v$) times the income left after purchasing the subsistence levels of housing and nonhousing goods and services.

Hence, we have derived expressions for the optimal quantities to consume of X_1 and X_2 in order to maximize utility. However, to continue designing the variable compensation program we need to solve what mathematicians call the dual problem to the constrained maximization problem above. Solving this dual problem will provide the implied expenditure function for our typical servicemember. Walter Nicholson defines the concept of "duality" in this manner:

Any constrained maximization problem has associated with it a dual problem in constrained minimization that focuses attention on the constraints in the original ("primal") problem.... The dual problem for the consumer is to minimize the expenditure needed to achieve a given level of utility.²

Given this definition of duality, the dual cost minimization problem to our original utility maximization problem above is:

$$\text{minimize } P_1X_1 + P_2X_2 = M$$

$$\text{subject to } U(X_1, X_2) = U^0.$$

The notation is the same with the exception that utility is fixed at the maximum level of utility achieved in the original problem above (U^0). Solving this cost minimization problem with the Lagrange multiplier method discussed above gives us the Lagrangian expression:

$$L = P_1X_1 + P_2X_2 + z(U^0 - (X_1 - B_1)^v(X_2 - B_2)^{1-v}).$$

Then, taking the derivative of L with respect to X_1 , X_2 , and z yields the following necessary first-order conditions:

$$L_1 = P_1 - zv(X_1 - B_1)^{v-1}(X_2 - B_2)^{1-v} = 0$$

$$L_2 = P_2 - z(1-v)(X_1 - B_1)^v(X_2 - B_2)^{-v} = 0$$

$$L_z = U^0 - (X_1 - B_1)^v(X_2 - B_2)^{1-v} = 0.$$

Solving the three simultaneous equations above yields:

$$X_1 = B_1 + [vP_2/((1-v)P_1)]^{1-v}U^0$$

and

$$X_2 = B_2 + [vP_2/((1-v)P_1)]^{-v}U^0$$

as the optimal consumption amounts of housing and nonhousing goods and services to minimize expenditures in obtaining the level U^0 of utility.⁴

An expenditure function represents the minimum expenditure needed to achieve a given level of utility.

Thus, it is a function of prices and the given utility level. The expenditure function for our typical servicemember is expressed as:

$$E(P_1, P_2, U^0) = P_1 X_1(P_1, P_2, U^0) + P_2 X_2(P_1, P_2, U^0).$$

Substituting for X_1 and X_2 from the cost minimization solution above yields:

$$E(P_1, P_2, U^0) = P_1 B_1 + P_2 B_2 + U^0 P_1^\gamma P_2^{1-\gamma} (1 - \gamma)^{\gamma-1}.$$

So far we have specified a Stone-Geary utility function for a typical servicemember and have derived the implied cost minimizing expenditure function. Given a specific utility level and prices of housing and nonhousing goods and services, the expenditure function for our typical servicemember makes the following statement. A servicemember will spend enough to purchase the subsistence levels of housing and nonhousing goods and services ($P_1 B_1 + P_2 B_2$) plus an additional amount. This additional amount is a function of the specified utility level, prices, and the marginal budget shares of housing and nonhousing goods and services ($U^0 P_1^\gamma P_2^{1-\gamma} (1 - \gamma)^{\gamma-1}$).

The concept of the variable compensation program is to derive an expenditure function for a typical servicemember that provides an equitable paid compensation value for a particular paygrade in an assignment location. This expenditure function is:

$$E(P_1, P_2, U^0) = P_1 B_1 + P_2 B_2 + U^0 P_1^\gamma P_2^{1-\gamma} (1 - \gamma)^{\gamma-1}.$$

Section 3.3 of this thesis discusses determining appropriate levels of U^0 for the various paygrades. Section 3.4 discusses the availability of the various types of data the variable compensation program requires. The other terms in the expenditure function we need to know are the values for B_1 , B_2 , and v since $(1-v)$ follows directly from v . Thus, the terms U^0 , P_1 , and P_2 will be given. However, the terms B_1 , B_2 , and v are unknown and must therefore be estimated. Once we have estimated values for B_1 , B_2 , and v we can plug these values into the expenditure function along with the given values of U^0 , P_1 , and P_2 for a specific paygrade in a specific assignment location. Then we can solve the expenditure function for the appropriate level of expenditure necessary to achieve the given utility level in that location. The next section discusses the procedures of estimating the parameters B_1 , B_2 , and v (the subsistence levels of housing, nonhousing goods and services, and the marginal budget share of housing, respectively).

3.2 ESTIMATING THE PARAMETERS

This section describes the process of estimating the three parameters B_1 , B_2 , and v in the expenditure function for our typical servicemember which was derived in Section 3.1. First, the concept of duality between utility maximization and cost minimization problems is addressed again. Then, the econometric specification for the amount spent on housing (X_1) and nonhousing goods and services (X_2) is derived. Finally, Stone's estimation methodology is described in detail and other estimation methods are highlighted.

Section 3.1 discussed the concept of duality in which for every "primal" constrained utility maximization problem there exists a dual cost minimization problem that is constrained by the level of utility achieved in the primal problem. Chiang, in his textbook, Fundamental Methods of Mathematical Economics, further states that:

In fact, since the optimal values of the objective functions in the primal and in the dual are always identical, we now have the option of picking the easier of the two programs to work with;...it is always possible also to translate the solution values of the dual-program variables into those of the primal-program variables and vice versa.®

Thus, the optimal values of X_1 and X_2 for the primal utility maximization and dual cost minimization problems in Section 3.1 are the same. Accordingly, the expenditure function in the dual cost minimization problem, $E(P_1, P_2, U^0) = P_1 X_1(P_1, P_2, U^0) + P_2 X_2(P_1, P_2, U^0)$ yields the same solution as the summation of expenditures in the primal utility maximization problem, $M = P_1 X_1(P_1, P_2, M) + P_2 X_2(P_1, P_2, M)$.

As a result, either expression for the optimal values of X_1 and X_2 can be used to estimate the three parameters B_1 , B_2 , and v . I use the Marshallian demand equations $X_1(P_1, P_2, M)$ and $X_2(P_1, P_2, M)$ because the exogenous income variable is easier to measure.

Regression or econometric analysis is necessary to estimate the parameters B_1 , B_2 , and v . This is accomplished by examining how these three parameters are related to other known variables. The first step in estimating these parameters is to derive the econometric specification for the amount of money spent on housing (X_1) and nonhousing goods and services (X_2). In Section 3.1 the optimal amounts of housing and nonhousing goods and services were

$$X_1(P_1, P_2, M) = B_1 + v(M - P_1 B_1 - P_2 B_2)/P_1$$

and

$$X_2(P_1, P_2, M) = B_2 + (1 - v)(M - P_1 B_1 - P_2 B_2)/P_2.$$

Hence, the amount of money spent on housing is $P_1X_1 = P_1B_1 + v(M - P_1B_1 - P_2B_2)$. Similarly, the amount spent on nonhousing goods and services is $P_2X_2 = P_2B_2 + (1 - v)[M - P_1B_1 - P_2B_2]$. That is, the amount spent on either housing or nonhousing goods and services is the amount spent on its subsistence level plus its marginal budget share times the amount of income available after the subsistence levels of both commodities are purchased.

Therefore, the general expression for the amount spent on either type of commodity is $P_iX_i = v_iM + \sum_j v_{ij}P_jB_j$. In this expression $i = 1, 2$ and $j = 1, 2$ where 1 represents housing and 2 represents nonhousing goods and services. M is the given level of income. P_i is the price of the specified commodity. X_i is the quantity of the commodity. B_i is the subsistence level of it, and v_i is its marginal budget share ($v_1 = v$, $v_2 = 1 - v$). On the other hand, P_j , X_j , B_j , and v_j are the respective values of the other type commodity. The term v_{ij} equals $-v_iB_j$ if i is not equal to j and it equals $(1 - v_i)B_i$ if i equals j . The summation sign \sum_j sums the expression $v_{ij}P_jB_j$ for both values of j . Solving this general expression for $i = 1$ to derive the amount spent on housing yields:
 $P_1X_1 = vM + (1 - v)P_1B_1 - vP_2B_2$. This is the same as

$P_1X_1 = P_1B_1 + v(M - P_1B_1 - P_2B_2)$, which was derived earlier in this section as the amount spent on housing.

Now, the econometric specification of the general equation for the amount spent on either housing or nonhousing goods and services is

$P_{it}X_{it} = v_iM_t + \sum_j v_{ij}P_{jt} + u_{it}$. The letter t represents the time period for the exogenous (given) parameters price (P), quantity (X), and income (M). The letters $i = 1, 2$ and $j = 1, 2$ where 1 and 2 represent housing and nonhousing goods and services, respectively. In the terminology of regression analysis, the term $P_{it}X_{it}$ is the dependent variable of the amount spent on the specified commodity. M_t and P_{jt} are the independent variables or regressors of income and the price of the j , or other type, commodity. The term u_{it} is the error term in estimating the amount spent on the specified commodity. The two major reasons the error term is present in the econometric specification are measurement and stochastic errors.* Measurement error includes the error in collecting the data on servicemembers' expenditures due to such mistakes as reporting and recording inaccuracies. Stochastic error includes the combined errors of omitting independent variables in the econometric specification that influence the dependent variable. Thus, the error term u_{it} is the sum of measurement and stochastic errors in estimating the

dependent variable of the amount spent on either composite commodity. The terms v_1 and v_{1j} are the unknown coefficients of the respective regressors M_t and P_{jt} . The estimates for v_1 and v_{1j} will provide us with estimates for v , B_1 , and B_2 .

Therefore, the econometric specification states that the amount spent by servicemembers on housing and nonhousing goods and services for a particular period of time t is known, along with the respective prices facing servicemembers, the quantities they consume of each category, and their available incomes. This given data is then used to estimate the unknown parameters of v_1 and v_{1j} .

Generally, the most basic technique used in estimating parameters is the method of ordinary least squares. Ordinary least squares makes the following assumptions about the error terms. (1) The expected value of each error term is zero. (2) The variance of each error term is constant. (3) The covariance (a measure of positive, negative, or zero relation between two variables) of each error term with every other error term is zero. In other words, the error terms are independent of each other. The objective of ordinary least squares is to minimize the summation of the error terms squared. In this problem, the error term $u_{1t} = P_{1t}X_{1t} - v_1M_t - \sum_j v_{1j}P_{jt}$. Thus, the ordinary least

squares method would minimize the summation of $[P_{1t}X_{1t} - v_1M_t - \sum_j v_{1j}P_{jt}]^2$ for all values of i and j ($i = 1, 2, j = 1, 2$).

It is easier to derive the equations for the estimated parameters by rewriting the econometric specification in the following matrix terms. Let $P_{1t}X_{1t} = Y_t$ which is the (2×1) matrix of $[P_{1t}X_{1t} \ P_{2t}X_{2t}]'$. Let X_t be the (2×6) matrix of

$\begin{bmatrix} M_t & P_{1t} & P_{2t} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_t & P_{1t} & P_{2t} \end{bmatrix}$. Let G be the (6×1) matrix of $[v_1 \ v_{11} \ v_{12} \ v_2 \ v_{21} \ v_{22}]'$.

Finally, let $u_t =$ the (2×1) matrix of $[u_{1t} \ u_{2t}]'$. Now, the econometric specification

$P_{1t}X_{1t} = v_1M_t + \sum_j v_{1j}P_{jt} + u_{1t}$ is rewritten as $Y_t = X_tG + u_t$. The ordinary least squares method of

estimation is to minimize the sum of squared errors.

The error term for time period t is u_t , which is equal to $Y_t - X_tG$. Hence, using the above matrix notation,

the sum of squared errors over all time periods

($t=1, \dots, k$) is written as $(Y - XG)'(Y - XG)$.

Y is the $(2k \times 1)$ column vector $[P_{1t}X_{1t} \ P_{2t}X_{2t}]'$ for $t=1, \dots, k$.

X is the $(2k \times 6)$ matrix $\begin{bmatrix} M_t & P_{1t} & P_{2t} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_t & P_{1t} & P_{2t} \end{bmatrix}$

for $t=1, \dots, k$.

The term u_t is the $(2k \times 1)$ column vector $[u_{1t} \ u_{2t}]'$ for $t=1, \dots, k$.

Let

$$D = (Y - XG)'(Y - XG).$$

Then, using rules of matrix multiplication:

$$D = Y'Y - Y'XG - (XG)'Y + (XG)'XG.$$

Since $Y'XG$ is a (1×1) matrix, then

$$Y'XG = (XG)'Y \text{ and}$$

$$D = Y'Y - 2G'X'Y + G'X'XG.$$

Thus, the problem is to:

$$\text{minimize } D = Y'Y - 2G'X'Y + G'X'XG.$$

Taking the derivative of D with respect to G and setting it equal to zero yields:

$$dD/dG = -2X'Y + 2X'XG = 0.$$

Since

$$-X'Y + X'XG = 0$$

then

$$\hat{G} = [X'X]^{-1}X'Y$$

which is the matrix of estimates for the parameters.¹⁰

However, if the assumption that the error terms are independent is dropped, then the matrix of parameter estimates becomes $\hat{G} = [X'A^{-1}X]^{-1}X'A^{-1}Y$ where A is the covariance matrix of the error terms. The covariance matrix is the matrix of the covariances of each error term with itself and every other error term.

This is important because an essential feature of the econometric specification

$P_{1t}X_{1t} = v_1M_t + \sum_j v_{1j}P_{jt} + u_{1t}$ is that the covariance matrix is singular. The significance of the covariance matrix being singular is that it cannot be inverted (A^{-1}) which is a requirement to solve for the estimators when the error terms are not independent.

To prove that the covariance matrix is singular first apply the adding up constraint that

$$P_{1t}X_{1t} + P_{2t}X_{2t} = M_t. \quad (1) \quad \text{Since,}$$

$$P_{1t}X_{1t} = v_1M_t + (1 - v_1)P_{1t}B_1 - v_1P_{2t}B_2 + u_{1t}$$

and

$$P_{2t}X_{2t} = (1 - v_1)M_t - (1 - v_1)P_{1t}B_1 + v_1P_{2t}B_2 + u_{2t}$$

then

$$M_t = M_t(1 - v_1 + v_1) + P_{1t}B_1(1 - v_1 - 1 + v_1) +$$

$$P_{2t}B_2(v_1 - v_1) + u_{1t} + u_{2t}.$$

Thus,

$$M_t = M_t + u_{1t} + u_{2t}$$

and

$$0 = u_{1t} + u_{2t}.$$

The error terms u_{1t} and u_{2t} sum to zero. By definition, the covariance matrix is equal to the expected value of the column vector of error terms u_t multiplied by its transpose u_t' . This is expressed as $A = E(u_t u_t')$ where A is the covariance matrix, E stands for expected value, and u_t is the column vector of error

terms. If A is multiplied by a row vector of all ones (designated by i') this is equivalent to $i'A = i'E(u_t u_t') = E(i'u_t u_t')$. Since, $i'u_t$ is the same as summing all the error terms, then $i'u_t = 0$.

Accordingly, $i'A = 0$ and the inverse of A is undefined. Thus, the covariance matrix cannot be inverted and it is singular.

Given that the covariance matrix A is singular for this particular econometric specification, the equation $\hat{G} = [X'A^{-1}X]^{-1}X'A^{-1}Y$ can no longer be used to solve for the estimates of the parameters. Hence, another method is required to solve for the estimates of the unknown parameters.

One way of solving this problem is a technique by Richard Stone. This method is an iterative ordinary least squares approach.¹² Stone's technique minimizes the sum of squared errors for

$P_{1t}X_{1t} = v_1M_t + \sum_j v_{1j}P_{jt} + u_{1t}$ as follows. The restrictions on v_{1j} are that v_{1j} equals $-v_1B_j$ if i is not equal to j and v_{1j} equals $(1 - v_1)B_1$ if i equals j . First set $B_1 = B_2 = 0$. This leaves

$$P_{1t}X_{1t} = v_1M_t + u_{1t}.$$

The estimates v_1 can be derived by regressing income against expenditure for a specified commodity. These estimates of v_1 are denoted $v_1^{(0)}$. These estimates for

v_1 can be used to derive estimates for B_1 and B_2 in the following manner.

$$P_{1t}X_{1t} - v_1^{(0)}M_t = \sum_j v_{1j}P_{jt} + u_{1t}.$$

Given the definition of v_{1j} , then

$$B_1 = v_{11}/(1-v_1)$$

and

$$v_{1j} = -v_1 v_{jj}/(1-v_{jj}).$$

Thus,

$$P_{1t}X_{1t} - v_1^{(0)}M_t = -\sum_j [v_1^{(0)}/(1 - v_{jj}^{(0)})] v_{1j}P_{jt} + u_{1t}.$$

Every variable is given except for v_{jj} . Regression analysis can provide an estimate for v_{jj} . This also gives an estimate for B_1 since $B_1 = v_{11}/(1-v_1)$. These estimates for B_1 are denoted as $B_1^{(0)}$ and can be used to derive new estimates for v_1 . This iterative least squares approach of using estimates for v_1 to derive estimates for B_1 and then using the estimates for B_1 to derive new estimates for v_1 , etc. is repeated until convergence is reached. The main drawback of Stone's method is the length of time it takes to reach convergence.

Other more complex methods of treating this problem of a singular covariance matrix for this econometric specification include the Gradient and Newton-Raphson technique and the ridge-walking algorithm. Formal treatments of these techniques are found in chapter 4 of Deaton [1975] listed in the bibliography. Deaton also

discusses the technique of maximum likelihood estimation for solving this problem in Deaton [1974]. The purpose of maximum likelihood estimation is to find the estimates of the parameters that are "most likely to generate the observed sample."¹³ This technique first states the likelihood function for a sample and then maximizes it to find the best estimators of the parameters.

Whatever technique is used, the expenditure function for a typical servicemember,

$$E(P_1, P_2, U^0) = P_1 B_1 + P_2 B_2 + U^0 P_1^v P_2^{1-v} (1-v)^{v-1},$$

now has estimates for the parameters P_1 , B_2 , v , and $(1-v)$. The next section discusses how the Department of Defense can determine the appropriate utility levels (U^0) for the various paygrades.

3.3 DETERMINING UTILITY LEVELS FOR THE DIFFERENT PAYGRADES

This section discusses a method for determining appropriate utility levels for the various paygrades in the armed services. These levels of utility (U^0) can then be used in the expenditure function for a typical servicemember which was derived in Section 3.1 in order to derive the appropriate compensation levels. Basically, these utility levels will be a reflection of how happy the Department of Defense (DoD) wants the various ranks of servicemembers to be in order to retain a sufficient number of personnel in the armed services, while staying within the defense budget. Essentially, the DoD already calculates the desired utility levels for servicemembers in the various paygrades through its annual budget process for military compensation. DoD analyzes the effects of pay increases and decreases through the Annualized Cost of Leaving (ACOL) model for servicemembers. First, an overview of the existing ACOL model for servicemembers is presented. Then, the ability of the ACOL model to project retention rates of servicemembers and the corresponding military career force structure and utility levels for each paygrade is discussed. Finally, some shortfalls of the static nature of the typical servicemember's expenditure

function in this military compensation model are examined.

A 1986 special study by the Congressional Budget Office of the United States summarized the ACOL model in the following manner:

The ACOL model relates the reenlistment decision of a service member to a comparison between the present value of the total pecuniary and nonpecuniary returns to leaving military service immediately, the total returns to staying for the optimal number of additional years and then leaving the military, and other factors such as civilian unemployment rates and members' taste for military service. The optimal additional career is the one that maximizes the net present value of the military career. All things being equal, real increases (decreases) in basic pay bolster (lower) the cost of leaving military service immediately, thus raising (lowering) the reenlistment rates. The change in reenlistment rates can be used to derive new continuation rates which, in turn, lead to a new career force.¹⁴

Fundamentally, the ACOL model can project changes in the retention rates of servicemembers based on changes in several variables including their income.

A description of the mechanics of the ACOL model is presented below in a manner closely similar to that in John Warner and Matthew Goldberg's discussion of the ACOL model in their February 1984 article in The Review of Economics and Statistics which is listed in the bibliography. The utility to a servicemember of staying in the military for n more years is designated as U_n . U_n is the sum of the present value of military pay and a monetary equivalent for the member's taste for military life over the period of n years plus the sum of the present value of military retirement pay, civilian earnings, and a monetary equivalent for the member's taste for the aspects of civilian life over the period $(n + 1)$ years through his life expectancy. Hence, U_n is written as:

$$U_n = \sum_{j=1}^s (M_j + V_m) d^j + \sum_{j=n+1}^T (R_{jn} + W_{jn} + V_c) d^j$$

where

M_j = the individual's expected military pay in each future year of service, $j=1, \dots, s$, where s equals the maximum allowable additional years of service

$R_{j,n}$ = the yearly retired pay the individual will receive after n more years of service;

$j = n + 1, \dots, T$, where T is life expectancy

$W_{j,0}$ = the future civilian earnings stream the individual expects to receive if he leaves immediately, $j = 1, \dots, T$

$W_{j,n}$ = the future civilian earnings stream the individual expects to receive if he leaves after n more periods, $j = n + 1, \dots, T$

p = the individual's yearly discount rate

$d_j = (1/(1 + p))^j$ = the present value at the time of the reenlistment decision of a dollar received j years in the future, $j = 1, \dots, T$.

Let V_m and V_c denote the "taste factors" or annual monetary equivalents of the non-monetary aspects of military and civilian life, respectively.¹³

The utility to a servicemember of leaving the military immediately is designated as U_c . U_c is the present value of the sum of expected civilian earnings and the member's taste for the aspects of civilian life over the rest of his expected life. Therefore, U_c is written as:

$$U_c = \sum_{j=1}^T (W_{j,0} + V_c) d_j.$$

The cost to a member of leaving the military is defined as the difference between the present value of staying in the military for n more years (U_n) and

leaving the military immediately (U_e) and is designated as C_n . This is expressed as

$$C_n = U_n - U_e = \sum_{j=n+1}^{\infty} (M_j + V_m)d_j + \sum_{j=n+1}^{\infty} (R_{jn} + W_{jn} + V_e)d_j - \sum_{j=1}^{\infty} (W_{j0} + V_e)d_j.$$

Accordingly, the situation in which a servicemember will stay in the military is when the cost of leaving is nonnegative or $C_n = U_n - U_e > 0$. This can be expressed as:

$$C_n = \sum_{j=n+1}^{\infty} M_j d_j + \sum_{j=n+1}^{\infty} (R_{jn} + W_{jn})d_j - \sum_{j=1}^{\infty} (W_{j0})d_j > \sum_{j=1}^{\infty} (V_e - V_m)d_j.$$

Furthermore, if C_n is divided by the sum of the term d_j , then the cost to a servicemember of leaving the military is annualized and is designated by A_n . Likewise, if each side of the equation above for $C_n > 0$ is divided by the sum of the term d_j , then the condition for a servicemember to stay in the military is rewritten as: $A_n = C_n / \sum_{j=1}^{\infty} d_j > (V_e - V_m)$. A_n is the annualized cost of leaving or ACOL value for an individual servicemember and $(V_e - V_m)$ is the individual's net preference for civilian life. Therefore, this equation states that a servicemember will stay in the military for n more years if the annualized cost of leaving is greater than the member's net taste for the aspects of civilian life.

With this background of what a servicemember's annualized cost of leaving represents, the next step is to summarize how the ACOL model can estimate retention

rates of servicemembers. As described above, the ACOL value for an individual servicemember is the annualized value of the difference in expected pay between staying in the military for n more years and leaving the military immediately. The ACOL values for the various paygrades could be calculated from data on the present military force structure available through the Defense Manpower Data Center.¹⁴ Moreover, the Defense Manpower Data Center could also provide the current retention rates of the various paygrades. These retention rates can be used to calculate the current career force structure of the military.

Now, the ACOL model can estimate retention rates in the following manner. The primary regression equation in the ACOL model is $\ln(R/(1 - R)) = a + bACOL + u$ where: \ln is natural logarithm, R is the retention rate by paygrade, a is a constant parameter, b is the coefficient of the independent variable ACOL, and u is a random error term.¹⁷ Given the current values of ACOL and R , ordinary least squares regression analysis can estimate values for a and b . Then, if the DoD experiments with different pay levels for the various paygrades, new ACOL values can be calculated. The new retention rates are derived by substituting the new ACOL values and estimates of a and b into the equation above. These new retention rates can then be used to calculate

the new projected career force structure of the military. Thus, the DoD can use the ACOL model to project what the retention rates of servicemembers and the corresponding military force structures will be using various options of increases or decreases in compensation levels.

The point behind this overview and discussion of the ACOL model is that it is already used by the DoD and it can be used to derive the utility levels of the various paygrades needed in the development of the variable compensation program in this thesis. In the ACOL model presented above, the annualized utility received by a servicemember for staying in the military n more years is

$$A_n = [\sum_{j=n}^{\infty} M_j d^j + \sum_{j=n+1}^{\infty} (R_j n + W_j n) d^j - \sum_{j=1}^{\infty} (W_j n) d^j] / \sum_{j=1}^{\infty} d^j.$$

If only the term $\sum_{j=n}^{\infty} M_j d^j / \sum_{j=1}^{\infty} d^j$ is calculated, this results in the portion of annualized utility that is attributable to what can be bought with military pay. In this thesis, a servicemember only derives utility from consumption of the composite goods housing or nonhousing goods and services. Therefore, the process of the ACOL model yields the annualized utility levels by paygrade that are attributable to the potential consumption of housing and nonhousing goods and services with military pay. As a result, these values of utility

can be used as estimates of U^0 in the expenditure function of a typical servicemember derived in Section 3.1.

It should be acknowledged here that the variable compensation program in this thesis derives a static expenditure function for a typical servicemember which is based on a given level of utility for the present year. This static model solves the problem of how a servicemember should allocate a given level of total expenditure or income among the available goods and services to achieve a certain level of utility. In essence, it assumes away problems that affect decisions regarding the "timing of purchases" such as changes in: interest rates, expected levels of income and inflation, age, and labor supply constraints.¹⁰ Furthermore, it does not address the element of savings.

A key shortfall of a model that does not include the element of savings is that over a typical person's life cycle there is a general pattern of saving and dissaving that will greatly influence the person's consumption pattern. People save for a number of reasons. Cash outlays for college tuition and down payments on automobiles and homes often require months or years of prior savings. The uncertainty of future employment or anticipated consumption requirements during retirement also induce many people to save during

their working years. Deaton and Muellbauer (1980) summarize a typical person's life cycle savings profile in the following way.

Household consumption needs are highest in the years of family formulation while income flows are lowest after retirement. Consequently, the early years of the life cycle are likely to be ones of low-saving or dissaving. Later on, savings become positive reaching a peak sometime before retirement, after which point there is dissaving. Consequently, the aggregate relationship between consumption and income depends upon both the age structure of the population and on the relative incomes of different age groups within it.¹⁹

Thus, the total impact that savings would have on a typical servicemember's consumption function would depend on the age structure of the military forces and the relative pay levels of the different age groups. An additional consideration is that the military's relatively generous retirement system may induce members of all ages to save less of their incomes for their retirement than people of other professions.

Two possible ways of incorporating savings into this static variable compensation program are to include it as a consumption good or treat it as expenditures that are deferred until a person retires. The first method would basically add another consumption good (X_3) to the typical servicemember's utility function that represents saving as a certain percentage of income. The main problem with this approach is that most people do not save money just for the sake of saving money. People save money in order to purchase goods and services at a later date. The second method of treating savings as expenditures that are deferred until one retires is a more accurate and complex approach. One way to accomplish this would be to analyze the average price levels of the locations where the majority of servicemembers retire. The amount of savings a typical servicemember would need to retire in these areas could be expressed as a function of the price levels of major categories of goods and services. Accordingly, the DoD, in its ACOL calculations for the proper utility levels, could either raise active duty or retirement pay levels to meet these savings requirements.

In sum, the variable compensation program in this thesis is a static model that does not contain the factor of savings. The expenditure function derived for a typical servicemember simply states what level of pay

is necessary to enable a member to reach a certain level of utility in a specified geographic location in the United States. The ACOL model described in this section is a dynamic model that embodies expected future military and civilian incomes in deriving these annualized levels of utility for each paygrade. The important point is that, together, the ACOL model and the expenditure function provide the means to derive the equitable levels of incomes for servicemembers of the same paygrade in different geographical locations, which is the objective of the variable compensation program. The element of savings could be added to this model of military compensation. One method would be to treat savings as a third consumption good in the expenditure function. A better, yet more involved, way would be to treat savings as expenditures deferred until after a member retires and incorporate these expenditures into the ACOL model calculations of the appropriate utility levels. However, for the purposes of this thesis, the principle of savings is not included in the design of the variable compensation program.

So far, Section 3.1 derived the expenditure function of a typical servicemember to be

$$E(P_1, P_2, U^0) = P_1 B_1 + P_2 B_2 + U^0 P_1^\nu P_2^{1-\nu} (1-\nu)^{\nu-1}.$$

Section 3.2 discussed techniques to derive estimates for the parameters B_1 , B_2 , ν , and $(1 - \nu)$. This section

addressed an existing process to determine estimates of U^o for the different paygrades in the military. The next section discusses the availability of the three different types of data required by the variable compensation program.

3.4 AVAILABILITY OF DATA

This section discusses the availability of the following three types of data which are needed to implement the variable compensation program. First, expenditure data is necessary to estimate the parameters in the typical servicemember's expenditure function which was described in Section 3.2. Secondly, data is needed to estimate servicemembers' retention rates as a function of utility which was described in Section 3.3. Finally, geographic price data for the composite commodities of housing and nonhousing goods and services is necessary to provide appropriate price levels for the typical servicemember's expenditure function. In regard to this third category of data, the Department of Defense (DoD) rationale for creating their own data base of geographic housing expenditure is discussed, along with potential sources of independent geographic price data.

The first category of data needed in this thesis is income and expenditure data to estimate the parameters of the typical servicemember's utility function. Once again, these parameters are B_1 (subsistence level of housing), B_2 (subsistence level of nonhousing goods and services), v (marginal budget share of housing) and $(1-v)$, (marginal budget share of nonhousing goods and

services). Similar studies have been conducted to estimate such parameters. One example is Deaton (1974). He used data from sources such as the United Kingdom Central Statistical Office: National Income and Expenditure, 1971 to estimate the marginal budget shares and subsistence levels of nine commodities for consumers in the United Kingdom from 1900-1970.²⁰ Additional studies using income and expenditure data include Ashenfelter and Heckman (1974) and Ghez and Becker (1975). The main source of data used by Ashenfelter and Heckman in estimating income and substitution effects in their model of family labor supply came from the 1960 Census of Population. The primary source of data used by Ghez and Becker in estimating "the elasticity of response of a family's consumption of goods to a change in" wage rates was the Bureau of Labor Statistics. Survey of Consumer Expenditures for 1960-1961.²¹ Furthermore, the Bureau of Labor Statistics. Consumer Expenditure Survey: Interview Survey, 1984 provides recent annual income and seven major expenditure data categories for families which are classified by nine separate family characteristics.²² These seven major categories of expenditure data are housing, food and alcoholic beverages, apparel, transportation, health care, entertainment, and other. These expenditure categories are classified by the following nine family

characteristics: quintiles of income before taxes, quintiles of total expenditures, income before taxes, age of reference person, size of consumer unit, composition of consumer unit, number of earners in a consumer unit, housing tenure and by race of reference person, and region of residence (northeast, midwest, south, west). Thus, data on various levels of consumers' incomes and expenditures is available. DoD could use this data to represent servicemembers' income and expenditures. Accordingly, DoD could use this data on income, expenditures and prices to estimate the four parameters of the typical servicemember's expenditure function, as described in Section 3.2.

The second type of data needed for the variable compensation program is data to estimate retention as a function of utility. This class of data is necessary to estimate the appropriate utility levels for the various paygrades in the military services. Warner and Goldberg (1984) used the annualized cost of leaving (ACOL) model in Section 3.3 to estimate these utility levels. The type of data used by Warner and Goldberg in their ACOL calculations for their model of Navy reenlistment behavior is presented next.

Basically, Warner and Goldberg needed data to calculate the annualized values of military active duty pay, retirement pay, and potential civilian earnings for

Navy enlisted personnel in their ACOL model. Their sources of data used in calculating these values were as follows (footnote 23 documents credit for all quoted material on these data sources): The Defense Manpower Data Center (DMDC) provided "background and military history data on 220,606 individuals" who decided between fiscal years 1974-1978 whether to remain in or leave the Navy after their first-term enlistment. This data plus military pay charts for each fiscal year were input into a "recursive dynamic programming algorithm" to calculate the expected military income for each individual. This algorithm included "average Navy promotion probabilities for each fiscal year" by respective paygrade and years of service. The data used in calculating civilian earnings came from "a set of longitudinal earnings data on 12,000 enlisted personnel who left the military in FY1969 after one term of service." This data was the basis for estimating civilian earnings "as a function of ... years of education, race, military entry test score, military occupation group, branch of military service, and linear and quadratic terms for years of post-service experience."²³ Thus, Warner and Goldberg mainly utilized data sources available from the DMDC and military pay charts to estimate the ACOL values for the individual Navy seamen. ACOL expresses the utility a servicemember receives from staying in the military in

terms of wages (military and potential civilian earnings). Therefore, these utility levels for the various paygrades can be used in the typical servicemember's expenditure function in the variable compensation program. This expenditure function will now express the compensation level in wages for the typical servicemember as a function of prices and fixed utility levels expressed in wages.

In addition to Warner and Goldberg (1984), there are several other examples of models expressing retention as a function of utility. Some of these are Gotz and McCall (1980, 1984), Congressional Budget Office (1986), Ward and Tan (1985), Hosek and Peterson (March 1985, May 1985), Hiller (1982), and Chow and Polich (1980). The vast majority of these studies obtained data used in calculating military pay values from the DMDC and/or DoD Personnel Surveys. Data used in calculating potential civilian earnings came from sources including the Bureau of Census Population Surveys, Bureau of Labor Statistics' data on unemployment rates and average hourly wages in manufacturing, and Rand's Medical Survey of Retired Military Personnel. Hence, data is available and has been used in several models that can provide estimates of servicemembers' utility levels which are needed in the variable compensation program.

The final type of data needed in the variable compensation program is geographic price data for the over 340 military assignment locations in the United States. First, the DoD rationale for creating their own geographic housing expenditure data base in lieu of using independent geographic price data is discussed. Then, problems with the current DoD method and possible sources of geographic price data, rather than expenditure data, are presented.

The DoD generates its own geographic housing expenditure data base in order to derive the variable housing allowance (VHA) rates in the current VHA program. This data is collected from a biennial sample survey of servicemembers' housing expenditures. DoD gives the following four main reasons for generating its own data base: (1) Civilian housing price indices are not available for many military assignment areas. (2) The price data that is available is often not current. (3) Civilian housing price data does not provide a true picture of servicemembers' housing costs. This claim is made based on the condition that servicemembers move more frequently than their civilian counterparts and thus their housing costs reflect inflationary effects more quickly. In addition, the transient nature of military life typically prevents servicemembers from enjoying the benefits of long-term rental agreements and

build-up of home equity. (4) Civilian price data is not available for the various paygrades in the military.²⁴ As a result, the current VHA program is based on housing expenditure data, self-reported by servicemembers, rather than independent civilian housing price data.

There are some problems with the current VHA rates being determined by self-reported expenditure data. To begin with, servicemembers know that under the current VHA program the more they spend on housing, the higher their VHA rates will be. Thus, this creates a potential tendency for servicemembers to spend more on housing than they otherwise would. Furthermore, DoD currently spends over \$250,000 to conduct a sample housing expenditure survey.²⁵ Finally, the current VHA calculations do not account for servicemembers who choose to make large down payments on their homes to lower their monthly mortgage payments or live further from the base to trade-off longer commuting times for more manageable housing expenses.²⁶ These efforts of servicemembers to reduce their monthly housing expenditures penalize them in the current VHA program.

Given that there is not an ideal set of price data on housing and nonhousing goods and services for every military assignment area in the United States, there is still potential for a better method of calculating VHA rates. Two promising sources of geographic price data

are the Bureau of Labor Statistics' Urban Family Budgets and the American Chamber of Commerce Researchers Association's Inter-City Cost of Living Indicators.

The Autumn 1980 Urban Family Budgets and Comparative Indexes for Selected Urban Areas provides price indexes for eight categories of consumption in 25 selected metropolitan areas and four nonmetropolitan areas.²⁷ Moreover, this data is provided for three different budget levels for a specifically defined standard four-person family. The three different budget levels are designated lower, intermediate, and higher. The key DoD complaint with this Bureau of Labor Statistics' data is that less than 20 percent of servicemembers are assigned in these 25 metropolitan areas.²⁸

Even though less than 20 percent of servicemembers are assigned in the 25 metropolitan areas covered by the Bureau of Labor Statistics' Urban Family Budgets, this data is still useful. It can be used in a model to estimate the cost of living variations among the different military assignment areas. One example of a multiple regression model of cost of living variations among different metropolitan areas is Haworth and Rasmussen (1973). Haworth and Rasmussen developed a model that "explains over 60 percent of variations in

the cost of living among metropolitan areas using three independent variables: city size, city form, and geographic region."²⁹ Their data sources for these three independent variables are as follows: city size - the 1970 census metropolitan area population, city form - number of physical barriers (water and mountains inside the residential limits as determined from large maps) that "severely limited a city's expansion through a 30 degree arc from the Central Business District (CBD)," and geographic region - a dummy variable for whether or not the metropolitan area is in the South (one if in the South, zero otherwise).³⁰ Their dependent variable, cost of living for a city, was taken from the Bureau of Labor Statistics, Three Budget Cost Estimates for an Urban Family of Four Persons, Spring 1970 for thirty-eight metropolitan areas.

Haworth and Rasmussen's model could be used to estimate the cost of living variations for the 25 metropolitan areas at three different budget levels using the 1980 Bureau of Labor Statistics' data. For example, (using Haworth and Rasmussen's notation) the econometric specification for the cost of living of a city is written as

$$C_i = b_1 \text{Pop}_i + b_2 \text{Form}_i + b_3 \text{Reg}_i + u_i.$$

C_i is the cost of living in city i . Pop_i is the population of city i . Form_i is the number of physical

barriers to expansion in city i . Reg_i is the dummy variable for whether or not the city is in the South (as defined by the Bureau of Labor Statistics). The term u_i is the error term. The terms b_1 , b_2 , and b_3 are the regression coefficients for the city's population, form, and region, respectively. This multiple regression analysis could provide equations for the cost of living as a function of these characteristics of a city. Furthermore, it could provide equations for a city's cost of housing and nonhousing goods and services as a function of the city's features. These econometric specifications would be the same as the one above except the dependent variable C_i would be replaced by H_i and N_i to represent the city's cost of housing and nonhousing goods and services, respectively. Also, additional independent variables could be added to Haworth and Rasmussen's model to try to improve the model. However, they found two additional variables to be statistically insignificant at the 95 percent level. These were the change in the population of a city between the period 1967 and 1970 and the total number of days the temperature in a city was over 90 degrees or below 32 degrees Fahrenheit.²¹

Data could be collected on the appropriate characteristics of the military assignment areas and plugged into these equations to estimate the appropriate

geographic cost of housing and nonhousing goods and services for each area. These geographic cost estimates could then be adjusted to serve as estimates of P_1 (geographic price of housing) and P_2 (geographic price of nonhousing goods and services) and be inserted into the typical servicemember's expenditure function.

Another possible source of geographic price data on housing and nonhousing goods and services is the quarterly publication of the American Chamber of Commerce Researchers Association's (ACCRA) Inter-City Cost of Living Indicators. The fourth quarter, 1985 Inter-City Cost of Living Indicators published cost of living indices for seven categories of consumption in 249 cities and 185 metropolitan or primary metropolitan statistical areas, and a separate price report for 59 commodities in the same 249 cities. The seven categories of cost of living indices are: groceries, housing, utilities, transportation, health care, miscellaneous goods and services, and an all items index. These indices measure relative price levels with 100 being the national average of the 249 cities. The 59 specific commodities which are individually priced in each city are all elements of the above six major consumption groups. The important point here is that if it is feasible for the ACCRA to collect geographic price data in 249 cities, it is also feasible for the DoD to

collect (or pay someone to collect) similar data for its over 340 military location areas in the United States.

According to the ACCRA, their cost of living indices are calculated in the following manner:

Weights assigned to relative costs are based on the latest government survey data on a mid-management executive family's pattern of expenditures for consumer goods and services. All items are priced at the local level by chamber of commerce research personnel at a specified time and by standard specifications and converted to an index by programmed computer processing.³²

Thus, the ACCRA publishes quarterly geographic cost of living data similar to the Bureau of Labor Statistics, except the ACCRA addresses only one family budget level and many more cities.

The DoD could first examine the 249 cities participating in the quarterly ACCRA survey to establish what percentage of servicemembers are assigned in these locations. For those military assignment areas in the United States which are not covered by the ACCRA survey, the DoD could attempt to collect similar data. It is possible that the cost of this data collection may prove to be more than the DoD's current VHA data collection expenses. If this is the case, then DoD could use the

cost of living data in a Haworth and Rasmussen type model to derive equations for the geographic cost of housing and nonhousing goods and services as described above with the Bureau of Labor Statistics' data. By either collecting geographic price data at those military assignment areas not covered in the Bureau of Labor Statistics or ACCRA's data or using models to estimate geographic cost of living variations for housing and nonhousing goods and services, the DoD can improve upon its current basis for VHA rates.

In sum, this section has identified and discussed existing data sources needed in the variable compensation program. The ongoing Bureau of Labor Statistics' Survey of Consumer Expenditures can provide a reasonable proxy for servicemembers' income and expenditure data. The DMDC, DoD personnel surveys, and various Bureau of Labor Statistics' data can provide extensive data for use in models estimating retention as a function of utility. Finally, the Bureau of Labor Statistics and the ACCRA publish cost of living data for various cities that provide a broad data set to use in multiple regression models to estimate equations for the relative geographic price levels of housing and nonhousing goods and services. The next section compares the advantages of the variable compensation

program outlined in this thesis with the current VHA program.

3.5 FOOTNOTES TO SECTION 3

¹ Eugene Silberberg, The Structure of Economics (New York: McGraw-Hill Book Company, 1978), p. 215.

² For a function $Y = f(X)$, the first derivative of Y with respect to X is written formally as:

$$dY/dX = df(X)/dX = \text{limit of } [f(X_0 + b) - f(X_0)]/b$$

as b tends toward 0. The letter d stands for derivative, X_0 is the original value of the variable X , and b is the change in X . The limit of the function as b tends toward 0 indicates we are interested only in very small changes in X .

³ The N th-derivative test for an extreme point of a function of one variable is detailed in Alpha C. Chiang, Fundamental Methods of Mathematical Economics, 3rd ed. (1967; rpt. New York: McGraw-Hill, 1984), pp. 266-267; hereafter cited as Chiang. Essentially, the first nonzero derivative of the function evaluated at the point satisfying the necessary first-order condition is the " N th derivative". If N is even and the derivative is negative, then the function is at a maximum. If N is even and the derivative is positive, the function is at a minimum. If N is odd, the function is at an inflection point.

⁴ See Appendix One for the sufficient second-order conditions for this utility maximization problem.

⁵ Walter Nicholson, Microeconomic Theory, 3rd ed. (1972; rpt. Chicago: The Dryden Press, 1985), pp. 47-48.

⁶ See Appendix Two for the sufficient second-order conditions for this cost minimization problem.

⁷ See Appendix Three for the derivation of the final expression of $E(P_1, P_2, U^0)$.

⁸ Chiang, p. 688.

⁹ Ronald J. Wonnacott and Thomas J. Wonnacott, Econometrics (New York: John Wiley & Sons, 1970), p. 17.

¹⁰ This matrix of parameter estimates, \hat{G} , results in a minimum value of the error term because $d^2D/dG^2 = 2xx$ which is greater than zero and fulfills the sufficient second-order condition for a minimum.

¹¹ The methodology of this proof is the same as in Angus Deaton, Models and Projections of Demand in Post-War Britain (London: Chapman and Hall, 1975), p. 36.

¹² Richard Stone, Mathematical Models of the Economy and Other Essays (London: Chapman and Hall, 1970), pp. 88-91.

¹³ Ronald J. Wonnacott and Thomas J. Wonnacott, Introductory Statistics for Business and Economics, 3rd ed. (1972; rpt. New York: John Wiley & Sons, 1984), pp. 510-511.

¹⁴ Congress of the United States, Congressional Budget Office, Elimination of Double Tax Benefits for Military Homeowners (March 1986), p. 30; hereafter cited as CBO.

¹⁵ John T. Warner and Matthew S. Goldberg, "The Influence of Non-Pecuniary Factors on Labor Supply: The Case of Navy Enlisted Personnel," The Review of Economics and Statistics, 66, No. 1 (1984), p. 27; hereafter cited as Warner and Goldberg (1984).

¹⁶ See Warner and Goldberg (1984), pp. 30-31 for a discussion of their techniques in calculating ACOL values.

¹⁷ CBO, p. 31.

¹⁸ Angus Deaton and John Muellbauer, Economics and Consumer Behavior (New York: Cambridge University Press, 1980), p. 97, 320; hereafter cited as Deaton and Muellbauer.

¹⁹ Deaton and Muellbauer, pp. 321.

²⁰ Angus S. Deaton, "The Analysis of Consumer Demand in the United Kingdom, 1900-1970," Econometrica, 42, No. 2 (1974), p. 354.

²¹ Gilbert R. Chaz and Gary S. Becker, The Allocation of Time and Goods Over the Life Cycle (New York: National Bureau of Economic Research, 1975), p. 133.

²² See U.S. Department of Labor, Bureau of Labor Statistics, Consumer Expenditure Survey: Interview Survey, 1984, Bulletin 2267, August 1986, pp. v and 5 for the nine family characteristics and the seven expenditure categories, respectively.

²³ See Warner and Goldberg (1984) p. 30 for the data sources and quoted material used in calculating military pay variables. See Warner and Goldberg (1984) p. 31 for the data sources and quoted material used in calculating civilian earnings.

²⁴ These four reasons are a summary of the DoD rationale for creating its own VHA expenditure data base in VHA Program, pp. IV-8, 9.

²⁵ VHA Program, p. IV-8.

²⁶ John Ginovsky, "Personnel Top Worry in Posture Statement," Air Force Times, 16 Feb. 1987, p. 6.

²⁷ See U.S. Department of Labor, Bureau of Labor Statistics News, USDL: 81-195, 22 April 1981, Autumn 1980 Urban Family Budgets and Comparative Indexes for Selected Urban Areas.

²⁸ Report of the President's Commission on Military Compensation (Washington, D.C.: Government Printing Office, April 1978), p. 138.

²⁹ C.T. Haworth and D.W. Rasmussen, "Determinants of Metropolitan Cost of Living Variations," Southern

Economic Journal, 40, No. 2 (1973), p. 183; hereafter cited as Haworth and Rasmussen (1973).

³⁰ Haworth and Rasmussen (1973), pp. 186-187.

³¹ Haworth and Rasmussen (1973), p. 188.

³² American Chamber of Commerce Researchers Association, Inter-City Cost of Living Indicators, Fourth Quarter, 1985, p. i.

4.0 COMPARISON OF THE CURRENT VHA PROGRAM AND THE VARIABLE COMPENSATION PROGRAM

This section highlights the key differences between the current VHA program and the variable compensation program discussed in this thesis. First, some apparent advantages of the variable compensation program are presented. Then, I conclude with some potential topics that might be considered in a more sophisticated analysis of military compensation.

The variable compensation program offers the following advantages over the current program: (1) The variable compensation program recognizes the geographic variability of the two composite commodities housing and nonhousing goods and services. The VHA program only acknowledges the geographic variability of housing. It assumes the cost of all other goods and services are identical throughout the United States. (2) The geographic data base for the variable compensation program's price variations can be based on either existing independent civilian price data or regression estimated prices using existing models of cost of living variations. The geographic data base for the VHA program is created using biennial sample surveys of servicemembers' self-reported housing expenditures. (3) Since the variable compensation program does not collect

servicemembers' expenditure data, it saves the cost of the biennial survey and the even more burdensome annual documentation of VHA offset for every servicemember which are both conducted in the VHA program. (4) Since the variable compensation program incorporates geographic price differentials into one lump sum of military compensation, it would require only one Congressional authorization for its requested budget. The current VHA program requires Congress to separately approve the requested basic military compensation and VHA budgets. Often times, this has resulted in erratic freezes or large increases in the VHA budget resulting in irregular financial burdens or hasty catch-up raises to servicemembers stationed in high cost areas. (5) The variable compensation program would distribute cost of living raises in a more tailored manner. That is, servicemembers living in higher cost growth areas need and would receive proportionately higher cost of living raises than those living in lower cost areas. The current VHA program restricts odd-numbered year allowance increases to be a standard percent across the board for all servicemembers. Altogether, the variable compensation program offers: a broader recognition of geographic price differences of goods and services throughout the United States, a more independent and accurate geographic price data base, elimination of the

administrative costs of sample surveys and servicemembers' VHA offset documentation, a simpler military compensation budget presented to the Congress, and a more equitable distribution of cost of living increases in military pay.

On the other hand, despite these advantages, a more detailed and sophisticated variable compensation program than the one presented in this thesis would address the following more complex issues: (1) The effects of military in-kind benefits (health care, commissary, etc.) on the typical servicemember's consumption patterns and utility function. (2) The effects of nonpecuniary elements on the utility function. These elements would include such considerations as the amenities living in San Diego may provide that living in Dayton does not, despite the higher cost of living in San Diego. (3) Extend beyond the two composite commodities case.

To begin with, the military services provide their members with certain in-kind benefits. Boadway and Wildasin (1984) define an in-kind benefit as a benefit received "in units of specific commodities such as food, housing or health care" as opposed to cash which could be used to buy the goods or services.¹ For example, the military services provide medical care to active duty members without charge in military hospitals and

clinics. Dependents of active duty members currently receive medical care without charge at military hospitals and clinics when space is available. When space is not available at military facilities, dependents are covered under the Civilian Health and Medical Program of the Uniformed Services (CHAMPUS). Active duty members with dependents do not pay premiums for the CHAMPUS insurance. However, they do pay annual deductibles (\$50 per individual/\$100 per family) and share the cost of the treatment received when the charges exceed the annual deductibles. Therefore, servicemembers and their families receive the majority of their health care in the form of an in-kind benefit, rather than receiving higher pay to spend on medical insurance premiums. Another example of an in-kind benefit is the subsidy of many of the items bought at the military commissaries and exchanges. On the average, a dollar spent on a given basket of items at commissaries and exchanges buys 24 percent more of these commodities than a dollar spent in commercial stores.² As a result, in both cases, a typical servicemember would tend to spend fewer dollars on consuming average amounts of these commodities than his civilian counterpart. Hence, in a more sophisticated analysis, the typical servicemember's expenditure function would account for the effects of these in-kind benefits.

Another possible topic of consideration in a more complex analysis is the effects of nonpecuniary elements on the typical servicemember's utility function. These elements include certain amenities such as year-round pleasant weather and recreational opportunities certain locations offer. For instance, a servicemember may prefer to live in San Diego, California because of the warm weather, pleasant scenery, and proximity to the beaches, despite a much higher than average cost of living. A servicemember may place such a high amount of utility on these amenities of living in San Diego that he would choose to live there rather than an area like Dayton, Ohio even if he was not compensated for the cost of living differential.

The variable compensation program in this thesis assumes that cost of living variations among cities are a function of more factors than simply the amenities a city offers. Otherwise, employees performing the same jobs in different cities would receive the same wages because the amenities enjoyed at a location would fully compensate a worker for the higher cost of living. However, earnings data compiled by the Bureau of Labor Statistics shows that hourly wages for manufacturing workers vary widely across the United States.² Thus, amenities are not the sole reason for cost of living variations.

The shortfall in the variable compensation program, regarding amenities, is that a servicemember is compensated with higher pay for the full cost of living variations among assignment locations in the United States. As a result, this program will tend to overcompensate servicemembers living in locations that are abundant with amenities. That is, a servicemember will be fully compensated for the higher cost of living while he is enjoying the extra benefits of the location's amenities. To more equitably compensate a servicemember for cost of living variations, he should not be compensated for the portion of the higher cost of living that is due to the amenities he will enjoy in his assignment location. A more sophisticated variable compensation program would account for the effects of amenities.

Another way of addressing the effects of amenities on the cost of living at each location is to allow servicemembers to choose their assignment locations. Under this scheme, servicemembers could choose where to work given an initial wage structure. Then, if too many members were choosing assignments in San Diego and too few were choosing Dayton, the Department of Defense could change the wage structure. Through a process of raising the wages in Dayton and lowering the wages in San Diego, the desired mix of people at the two

locations could be obtained. In fact, this process could be used to obtain the proper balance of military personnel at all locations. Of course, this approach of letting the market forces determine the levels of pay at the various assignment locations has serious shortcomings in the military environment. The diverse missions of the respective military services and overall national security requirements would certainly eliminate an option allowing complete freedom of assignment choices for servicemembers.

Finally, the expenditure function of a typical servicemember in this thesis recognizes only two composite commodities. A more involved analysis would expand the categories of consumption for a servicemember. Accordingly, increased estimates for the respective subsistence quantities and marginal budget shares of additional types of goods and services would improve the scope and detail of a typical servicemember's expenditure function.

In conclusion, the variable compensation program outlined in this thesis suggests several improvements in equitable compensation over the current VHA program. Nevertheless, this variable compensation program is based on a simplified analysis. A more intricate analysis would expand the scope of this variable compensation program to account for the important, yet

more complex, effects of in-kind benefits, the different amenities offered at different locations, and increased categories of consumption by servicemembers.

4.1 FOOTNOTES TO SECTION 4

¹ Robin W. Boadway and David E. Wildasin, Public Sector Economics, 2nd ed. (n.d.; rpt. Boston: Little, Brown & Company, 1984), p. 454.

² U.S. Department of the Air Force, Headquarters Air Force Accounting and Finance Center, Personal Statement of Military Compensation. Additional Considerations/Programs, Denver, Colorado, 31 Jan. 1987, p. 3.

³ U.S. Department of Labor, Bureau of Labor Statistics, Employment and Earnings, Vol. 33, No. 6, June 1986, pp. 134-138.

5.0 CONCLUSION

The primary goal of the current Department of Defense (DoD) variable housing allowance (VHA) program is to provide roughly the same compensation to servicemembers of the same paygrade and dependency status, regardless of their location of assignment in the United States. Two principal problems with the current VHA program are: (1) It assumes that only housing prices vary geographically in the United States. That is, the prices of all other goods and services are assumed to be identical throughout the United States. (2) The data base for geographic housing prices is based on DoD-conducted biennial sample surveys of servicemembers' self-reported housing expenditures which are uncontrolled for the quality of housing.

This thesis discusses the essential features of an alternative program which is called the variable compensation program. The sketch of this program begins with the assumption of a Stone-Geary utility function for a typical servicemember. This utility function specifies that a servicemember derives utility from the consumption of two composite commodities which are housing and nonhousing goods and services. The servicemember's implied expenditure function is derived and it is a function of the price of housing, the price

of nonhousing goods and services, and the utility level provided by the member's regular military compensation.

The parameters of this expenditure function are the subsistence quantities of housing and nonhousing goods and services as well as the respective marginal budget shares of each commodity. The procedures involved in estimating these parameters are presented. Stone's (1970) iterative ordinary least squares methodology and Deaton's (1975) maximum likelihood estimation technique are highlighted.

Likewise, the methodology of determining the appropriate levels of utility for the various DoD paygrades is examined. Warner and Goldberg's (1984) model of expressing retention as a function of utility by using the DoD's annualized cost of leaving (ACOL) model is emphasized.

The final segment of the outline of the variable compensation program is an analysis of data availability. The ongoing Bureau of Labor Statistics' (BLS) Survey of Consumer Expenditures provides a reasonable proxy for the income, expenditure and price data needed to estimate the parameters of the typical servicemember's utility function. Various sources of BLS data and Defense Manpower Data Center (DMDC) data are available to assist in expressing retention as a function of utility. The BLS Urban Family Budgets and

the American Chamber of Commerce Researchers Association's (ACCRA) Inter-City Cost of Living Indicators provide independent cost of living indices for 25 metropolitan areas and 249 cities, respectively. These data sets can be used in a cost of living variations multiple regression model such as Haworth and Rasmussen (1973). This model can estimate equations expressing cost of living, cost of housing and cost of nonhousing goods and services as functions of a city's size, the physical barriers to a city's expansion, and the geographic region of a city.

In sum, this basic variable compensation program provides numerous advantages to the current VHA program. The variable compensation program recognizes that prices of both housing and nonhousing goods and services vary throughout the United States. Furthermore, its data base for geographic price differences is based on regression estimated cost of living indices using independent civilian price data sets. Additionally, it possesses the potential for reduced administrative tasks by the DoD, a simpler compensation budget for presentation to the Congress, and a more equitable distribution of future cost of living raises.

All in all, the variable compensation program presents an outline of an alternative manner of addressing the issue of variable prices of goods and

services in the United States in regard to military compensation. This basic analysis could be expanded to incorporate the more complex effects of servicemembers' in-kind benefits, the amenities offered at assignment locations, and additional categories of consumption commodities.

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APPENDIX ONE

The utility maximization problem for a typical servicemember in Section 3.1 was to maximize the Lagrangian function

$$L(X_1, X_2, z) = (X_1 - B_1)^v (X_2 - B_2)^{1-v} + z(M - P_1 X_1 - P_2 X_2).$$

This can be rewritten as

$$L(X_1, X_2, z) = (X_1 - B_1)^v (X_2 - B_2)^{1-v} + z[M - g(X_1, X_2)]$$

where $g(X_1, X_2) = P_1 X_1 + P_2 X_2$.

The necessary first-order conditions

($L_1 = L_2 = L_z = 0$) were satisfied in solving for the optimal quantities of X_1 and X_2 (designated here as X_1^* and X_2^*) in Section 3.1. Thus, a positive value of the bordered Hessian determinant

$$\begin{vmatrix} 0 & g_1 & g_2 \\ g_1 & L_{11} & L_{12} \\ g_2 & L_{21} & L_{22} \end{vmatrix}$$
 is a sufficient second-order condition for the value of the function $L(X_1, X_2, z)$, which is the same as $U(X_1, X_2)$, to be a maximum at X_1^* and X_2^* .

In this utility maximization problem:

$P_1, P_2, z > 0$, $0 < v < 1$, $X_1 > B_1$, and $X_2 > B_2$.

Therefore:

$$g_1 = P_1 > 0.$$

$$g_2 = P_2 > 0.$$

$$L_1 = v(X_1 - B_1)^{v-1}(X_2 - B_2)^{1-v} - zP_1 = 0.$$

$$L_{11} = v(v-1)(X_1 - B_1)^{v-2}(X_2 - B_2)^{1-v} < 0.$$

$$L_{12} = v(1-v)(X_1 - B_1)^{v-1}(X_2 - B_2)^{-v} > 0.$$

$$L_2 = (1-v)(X_1 - B_1)^v(X_2 - B_2)^{-v} - zP_2 = 0.$$

$$L_{21} = v(1-v)(X_1 - B_1)^{v-1}(X_2 - B_2)^{-v} > 0.$$

$$L_{22} = -v(1-v)(X_1 - B_1)^v(X_2 - B_2)^{-(v+1)} < 0.$$

Now, the value of the bordered Hessian determinant, designated $|\bar{H}|$, is derived as follows.

$$|\bar{H}| = 0 - g_1 \begin{vmatrix} g_1 & g_2 \\ L_{21} & L_{22} \end{vmatrix} + g_2 \begin{vmatrix} g_1 & g_2 \\ L_{11} & L_{12} \end{vmatrix}.$$

$$|\bar{H}| = -g_1(g_1L_{22} - g_2L_{21}) + g_2(g_1L_{12} - g_2L_{11}).$$

Substituting in for the signs of the terms derived above yields:

$$|\bar{H}| = -[(+)(-) - (+)(+)] + [(+)(+) - (+)(-)].$$

$$|\bar{H}| = -[(-) - (+)] + [(+) - (-)].$$

$$|\bar{H}| = -[-] + [+] = [+] + [+].$$

Thus, the bordered Hessian determinant is positive and the optimal values X_1^* and X_2^* yield a maximum value of the utility function $U(X_1, X_2)$.

APPENDIX TWO

The cost minimization problem for a typical servicemember in section 3.1 was to minimize the Lagrangian function

$$L(X_1, X_2, z) = P_1 X_1 + P_2 X_2 + z[U^0 - (X_1 - B_1)^v (X_2 - B_2)^{1-v}].$$

This can be rewritten as

$$L(X_1, X_2, z) = P_1 X_1 + P_2 X_2 + z[U^0 - g(X_1, X_2)]$$

where $g(X_1, X_2) = (X_1 - B_1)^v (X_2 - B_2)^{1-v}$.

The necessary first-order conditions

($L_1 = L_2 = L_z = 0$) were satisfied in solving for the optimal quantities of X_1 and X_2 (designated here as X_1^* and X_2^*) in Section 3.1. Thus, a negative value of the bordered Hessian determinant

$$\begin{vmatrix} 0 & g_1 & g_2 \\ g_1 & L_{11} & L_{12} \\ g_2 & L_{21} & L_{22} \end{vmatrix}$$
 is a sufficient second-order condition for the value of the function $L(X_1, X_2, z)$, to be a minimum at X_1^* and X_2^* .

In this cost minimization problem:

$P_1, P_2, z > 0, 0 < v < 1, X_1 > B_1$, and $X_2 > B_2$.

Therefore:

$$g_1 = v(X_1 - B_1)^{v-1}(X_2 - B_2)^{1-v} > 0.$$

$$g_2 = (1 - v)(X_1 - B_1)^v(X_2 - B_2)^{-v} > 0.$$

$$L_z = P_1 - zv(X_1 - B_1)^{v-1}(X_2 - B_2)^{1-v} = 0.$$

$$L_{11} = -zv(v-1)(X_1 - B_1)^{v-2}(X_2 - B_2)^{1-v} > 0.$$

$$L_{12} = -zv(1-v)(X_1 - B_1)^{v-1}(X_2 - B_2)^{-v} < 0.$$

$$L_2 = P_2 - z(1-v)(X_1 - B_1)^v(X_2 - B_2)^{-v} = 0.$$

$$L_{22} = zv(1-v)(X_1 - B_1)^v(X_2 - B_2)^{-(v+1)} > 0.$$

$$L_{21} = -zv(1-v)(X_1 - B_1)^{v-1}(X_2 - B_2)^{-v} < 0.$$

Now, the value of the bordered Hessian determinant, designated $|\bar{H}|$, is derived as follows.

$$|\bar{H}| = 0 - g_1 \begin{vmatrix} g_1 & g_2 \\ L_{21} & L_{22} \end{vmatrix} + g_2 \begin{vmatrix} g_1 & g_2 \\ L_{11} & L_{12} \end{vmatrix}.$$

$$|\bar{H}| = -g_1(g_1L_{22} - g_2L_{21}) + g_2(g_1L_{12} - g_2L_{11}).$$

Substituting in for the signs of the terms derived above yields:

$$|\bar{H}| = -[(+)(+) - (+)(-)] + [(+)(-) - (+)(+)].$$

$$|\bar{H}| = -[(+) - (-)] + [(-) - (+)].$$

$$|\bar{H}| = -[+] + [-] = [-] + [-].$$

Thus, the bordered Hessian determinant is negative and the optimal values of X_1^* and X_2^* yield a minimum value of cost.

APPENDIX THREE

$$E(P_1, P_2, U^0) = P_1 X_1(P_1, P_2, U^0) + P_2 X_2(P_1, P_2, U^0).$$

$$X_1(P_1, P_2, U^0) = B_1 + [vP_2/((1-v)P_1)]^{1-v} U^0$$

and

$$X_2(P_1, P_2, U^0) = B_2 + [vP_2/((1-v)P_1)]^{-v} U^0.$$

$$\text{Let } E = E(P_1, P_2, U^0).$$

Therefore:

$$E = P_1 B_1 + P_1 [vP_2/((1-v)P_1)]^{1-v} U^0 + P_2 B_2 + P_2 [vP_2/((1-v)P_1)]^{-v} U^0.$$

$$E = P_1 B_1 + P_2 B_2 + U^0 [(vP_2/(1-v))^{1-v} P_1^v + (v/((1-v)P_1))^{-v} P_2^{1-v}].$$

$$E = P_1 B_1 + P_2 B_2 + U^0 [(vP_2/(1-v))^{1-v} P_1^v + ((1-v)P_1/v)^{-v} P_2^{1-v}].$$

$$E = P_1 B_1 + P_2 B_2 + U^0 P_1^v P_2^{1-v} [(v/(1-v))^{1-v} + ((1-v)/v)^{-v}].$$

$$E = P_1 B_1 + P_2 B_2 + U^0 P_1^v P_2^{1-v} [(v/(1-v))^{1-v} + (v/(1-v))^{-v}].$$

$$E = P_1 B_1 + P_2 B_2 + U^0 P_1^v P_2^{1-v} [v/(1-v)]^{-v} [v/(1-v) + 1].$$

$$E = P_1 B_1 + P_2 B_2 + U^0 P_1^v P_2^{1-v} [v/(1-v)]^{-v} [1/(1-v)].$$

$$E = P_1 B_1 + P_2 B_2 + U^0 P_1^v P_2^{1-v} v^{-v} (1-v)^{v-1}.$$